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Light-cone effect on higher-order clustering in redshift surveys

Takahiko Matsubara[1,2], Yasushi Suto[1,2], and István Szapudi[3]

[1] Department of Physics, University of Tokyo, Tokyo 113, Japan
[2] Research Center for the Early Universe (RESCEU), School of Science
University of Tokyo, Tokyo 113, Japan
[3] NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory
Batavia IL 60510-0500

e-mail: matsu@phys.s.u-tokyo.ac.jp, suto@phys.s.u-tokyo.ac.jp, szapudi@traviata.fnal.gov

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ABSTRACT

We have evaluated a systematic effect on counts-in-cells analysis of deep, wide-field galaxy catalogues induced by the evolution of clustering within the survey volume. A multiplicative correction factor is explicitly presented, which can be applied after the higher order correlation functions have been extracted in the usual way, without taking into account the evolution. The general theory of this effect combined with the ansatz describing the non-linear evolution of clustering in simulations enables us to estimate the magnitude of the correction factor in different cosmologies. In a series of numerical calculations assuming an array of cold dark matter models, it is found that, as long as galaxies are unbiased tracers of underlying density field, the effect is relatively small ($\simeq 10\%$) for the shallow surveys ($z < 0.2$), while it becomes significant (order of unity) in deep surveys ($z \sim 1$). Depending on the scales of interest, the required correction is comparable to or smaller than the expected errors of on-going wide-field galaxy surveys such as the SDSS and 2dF. Therefore at present, the effect has to be taken into account for high precision measurements at very small scales only, while in future deep surveys it amounts to a significant correction.

Subject headings: cosmology: theory - dark matter - large-scale structure of universe – galaxies: distances and redshifts

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1. Introduction

Cosmological observations are necessarily carried out on a null hypersurface or a light-cone. At low redshifts ($z < 0.1$), this can be regarded as to provide information on the constant-time hypersurface ($z = 0$) which is a quite conventional implicit approximation underlying cosmological studies using the galaxy redshift surveys. When the depth of the survey volume exceeds $z \sim 0.1$, however, this approximation breaks down, and one should simultaneously take account of the intrinsic evolution of galaxy clustering and the light-cone effect in addition to any other selection effect in interpreting the data. This is indeed the case for the on-going wide-field surveys of galaxies including 2dF (2-degree Field Survey) and SDSS (Sloan Digital Sky Survey).

To our knowledge, the first quantitative consideration of the light-cone effect is made by Nakamura, Matsubara, & Suto (1998) who derived the systematic bias in the estimate of $\beta \approx \Omega_0^{0.6}/b$ from magnitude-limited surveys of galaxies combining the cosmological redshift distortion effect (Matsubara & Suto 1996) and the evolution of galaxy clustering within the survey volume. In this paper, we examine the light-cone effect on higher-order statistics of galaxy clustering, considering counts-in-cells analysis specifically.

Let us consider first the higher-order statistics on the idealistic constant-time hypersurface. Denote the volume averaged N-th order correlation functions at a redshift z by $\overline{\xi}_N(R; z)$, where R is the comoving smoothing length, and introduce the normalized higher-order moments $S_N(R; z) \equiv \overline{\xi}_N(R; z)/[\overline{\xi}_2(R; z)]^{N-1}$. The hierarchical clustering ansatz states that $S_N(R; z)$ is constant and independent of the scale R. This is a good approximation in nonlinear regimes, although small but definite scale-dependence is clearly detected from N-body experiments (Lahav et al. 1993; Suto 1993; Matsubara & Suto 1994; Suto & Matsubara 1994; Jing & Börner 1997). In addition, perturbation theory predicts that $\overline{\xi}_N(R; z)$ evolves in proportion to $\left[\overline{\xi}_2(R; z)\right]^{N-1}$, and therefore $S_N(R; z)$ is independent of time, i.e. it is constant with respect to z.

The next section describes the general theory of the light cone effect on $S_N(R; z)$ defined above. Using the ansatz by Jain, Mo, & White (1995; hereafter JMW), §3 evaluates the appropriate correction in an array of cold dark matter (CDM) models. Finally, §4 summarizes the results and discusses the implications for redshift surveys.

2. Observing the higher-order moments on the light-cone

It is difficult to estimate $\overline{\xi}_N(R; z)$ observationally since z is changing over the volume of galaxy sample. While in principle one could measure the N-point functions on $z = const$ surfaces, in practice this would result in a diminished volume, thus a significant increase of the errors. Instead it is more practical to extract the following N-th order correlation functions averaged over

the volumes on the light-cone:

$$\overline{\xi}_N(R;<z_{\max}) \equiv \frac{\displaystyle\int_0^{z_{\max}} z^2dz\ w(z)\overline{\xi}_N(R;z)}{\displaystyle\int_0^{z_{\max}} z^2dz\ w(z)}. \tag{1}$$

In the above expression, we assume that the observation is performed with the fixed solid angle, and the sampling cells for the analysis are placed randomly in z-coordinate with $w(z)$ being their weighting function. If the cells were located randomly in the comoving coordinates, the volume element z^2dz should have been replaced by $d_A(z;\Omega_0,\lambda_0)^2c|dt/dz|dz$ (d_A is the angular diameter distance; see, Nakamura et al. 1998) and thus the procedure itself becomes dependent on adopted values of Ω_0 and λ_0. In principle $w(z)$ is an arbitrary function, and should be determined so as to maximize the signal-to-noise ratio given the selection function of individual observation. By $z_{\max}$ we denote the redshift corresponding to the depth of the survey. For a volume-limited sample, for instance, it is natural to set $w(z)=1$ and $z_{\max}$ as the maximum redshift of the sample. Similarly we define the (observable) higher-order moments averaged over the light-cone as

$$\overline{S_N}(R;<z_{\max}) \equiv \frac{\overline{\xi}_N(R;<z_{\max})}{\left[\overline{\xi}_2(R;<z_{\max})\right]^{N-1}}. \tag{2}$$

It is useful to introduce the function $G(z)$ which describes the evolution of the averaged two-point correlation function:

$$\overline{\xi}(R;z) = G(z)\overline{\xi}(R;0). \tag{3}$$

In linear regime, $G(z)$ is equivalent to $[D(z)/D(0)]^2$ where $D(z)=D(z;\Omega_0,\lambda_0)$ is the linear growth rate. Although the above relation (3) does not exactly hold in the nonlinear regime, several approximation formulae are derived in the literature, which empirically describe the evolution by allowing $G(z)$ depend on the scale R (see §3 for details).

Once we accept the evolution law (3), equation (2) is explicitly written as

$$\overline{S_N}(R;<z_{\max}) = \frac{\displaystyle\int_0^{z_{\max}} z^2dz\ w(z)S_N(R;z)\left\{G(z)\right\}^{N-1}\left[\int_0^{z_{\max}} z^2dz\ w(z)\right]^{N-2}}{\displaystyle\left[\int_0^{z_{\max}} z^2dz\ w(z)G(z)\right]^{N-1}}. \tag{4}$$

If $z_{\max} \ll 1$, the above expression is expanded in terms of $z_{\max}$ as follows:

$$\begin{aligned}\overline{S_N}(R;<z_{\max}) &= S_N(R;0) + \frac{3}{4}S_N'(0)z_{\max} \\ &+ \left[\frac{3}{160}(N-1)(N-2)S_N(0)G'(0)^2 + \frac{3}{80}(N-1)S_N'(0)G'(0) + \frac{3}{10}S_N''(0)\right]z_{\max}^2 \\ &+ \mathcal{O}(z_{\max}^3),\end{aligned} \tag{5}$$

where $S'_N(0)$ denotes $\partial S_N(R;z)/\partial z|_{z=0}$ and so on. The above expansion up to $\mathcal{O}(z_{\rm max}^2)$ is valid as long as the weighting function is well-approximated up to the same order:

$$w(z_{\rm max}) = w(0) + w'(0)z_{\rm max} + \frac{1}{2}w''(0)z_{\rm max}^2. \tag{6}$$

In other words, $z_{\rm max}$ should be set to be smaller than the effective window size of $w(z_{\rm max})$.

It is interesting to note that up to $\mathcal{O}(z_{\rm max}^2)$ equation (5) is independent of $w(z)$, and that $\mathcal{O}(z_{\rm max})$ term is determined only by $S'_N(0)$ independently of $G(z)$. Since $S'_N(0)$ is expected to vanish in linear theory (Fry 1984; Goroff et al. 1986; Bernardeau 1992), and shown to be relatively small even in quasi- and fully- nonlinear regimes (Bouchet et al. 1992; Lahav et al. 1993; Colombi, Bouchet, & Hernquist 1995; Szapudi et al. 1997), equation (5) implies that the light-cone effect is very small for 2dF and SDSS galaxy redshift surveys ($z_{\rm max} < 0.2$). It should be noted, however, that if galaxies are biased relative to the mass density field, $S'_N(0)$ may not be necessarily small. So any signal proportional to $z_{\rm max}$ provides a clear indication of the time-dependent biasing of galaxies (see e.g., Fry 1996; Mo & White 1996; Mo, Jing & White 1997).

3. Evaluating the light-cone effect: an example

In order to evaluate the effect of observational average on the light-cone, we assume that $S_N(R;z)$ does not evolve with z, i.e., $S_N(R;z) = S_N(R;0)$. As described above, this is a reasonable approximation as long as galaxies are unbiased tracers of underlying density field. If we introduce the measure of the light-cone effect:

$$\Delta_N(R;<z_{\rm max}) \equiv \frac{\overline{S_N}(R;<z_{\rm max})}{S_N(R;0)} - 1, \tag{7}$$

equations (4) and (5) with $S_N(R;z) = S_N(R;0)$ reduce to

$$\Delta_N(z_{\rm max}) = \frac{3}{160}(N-1)(N-2)G'(0)^2 z_{\rm max}^2 + \mathcal{O}(z_{\rm max}^3). \tag{8}$$

Note that $(1+\Delta_N)$ can be regarded as a correction factor as well, if one measures the S_N's *without* considering the evolution of clustering. This constitutes a simple and practical method, which we propose for future measurements, when compensation for the light cone effect is required.

To evaluate equation (4), we need a model for $G(z)$. For this purpose, we adopt the ansatz originally put forward by Hamilton et al. (1991) and improved later by Peacock & Dodds (1994, 1996) and JMW. To be specific, we apply the fitting formula by JMW which relates the evolved two-point correlation function $\overline{\xi}_E(R;z)$ with its linear counterpart $\overline{\xi}_L(R_0;z)$ as follows:

$$\overline{\xi}_E(R;z) = B(n)\,F[\overline{\xi}_L(R_0;z)/B(n)], \tag{9}$$

$$F(x) = \frac{x + 0.45x^2 - 0.02x^5 + 0.05x^6}{1 + 0.02x^3 + 0.003x^{9/2}}. \tag{10}$$

In the above equations, n denotes a power-law index of the power spectrum, $R_0 = [1+\overline{\xi}_E(z,R)]^{1/3}R$, and $B(n) = [(3+n)/3]^{0.8}$. JMW show that the above formula works reasonably well even for CDM models by replacing n by the effective spectral index evaluated at the scale which is just entering nonlinear regime. In general the resulting n depends on z, which we neglect below for simplicity; for galaxy surveys which we are primarily interested in, the z-dependence of n near $z=0$ is expected to be very small.

The inverse of equation (9) is formally written as $\overline{\xi}_L(R_0;z) = B(n)F^{-1}[\overline{\xi}_E(R;z)/B(n)]$ and JMW's empirical fit to $F^{-1}(y)$ is

$$F^{-1}(y) = y\left(\frac{1+0.036y^{1.93}+0.0001y^3}{1+1.75y-0.0015y^{3.63}+0.028y^4}\right)^{1/3}. \tag{11}$$

Then $\overline{\xi}_E(R;z)$ is expressed explicitly in terms of $\overline{\xi}_E(R;0)$:

$$\overline{\xi}_E(R;z) = B(n)F\left[\frac{D^2(z)}{D^2(0)}F^{-1}[\overline{\xi}_E(R,0)/B(n)]\right]. \tag{12}$$

Let us introduce a parameter $\alpha(R) \equiv F^{-1}\left[\overline{\xi}_E(R,0)/B(n)\right]$ which characterizes the variance on a scale R at $z=0$, and thus depends on Ω_0 and λ_0 through the shape of the fluctuation spectrum. Then the scale-dependent evolution factor $G(z) = G(R;z)$ in equation (3) is given by

$$G(R;z) \equiv \frac{\overline{\xi}_E(R;z)}{\overline{\xi}_E(R;0)} = \frac{1}{F(\alpha)}F\left[\frac{D^2(z)}{D^2(0)}\alpha\right]. \tag{13}$$

For the convenience of z-expansion, we calculate the derivatives of the above quantity at $z=0$:

$$\left.\frac{\partial G(R;z)}{\partial z}\right|_{z=0} = -2f_0\frac{\alpha F'(\alpha)}{F(\alpha)}, \tag{14}$$

$$\left.\frac{\partial^2 G(R;z)}{\partial z^2}\right|_{z=0} = 4f_0^2\frac{\alpha^2 F''(\alpha)}{F(\alpha)} + \left(2f_0^2+2f_0q_0+3\Omega_0\right)\frac{\alpha F'(\alpha)}{F(\alpha)}, \tag{15}$$

where $f_0 = d\ln D/da|_{z=0}$, $q_0 = \Omega_0/2 - \lambda_0$. The above expressions indicate how the light-cone effect depends on Ω_0 and λ_0 at $z_{\max} \ll 1$. Note, that they are involved in $\mathcal{O}(z_{\max}^2)$ term and thus do not contribute significantly at small z.

4. Results and conclusions

Using equations (4) and (13) and assuming $S_N(z) = S_N(0)$, we can evaluate the evolutionary effect on $\overline{S_N}(R;<z_{\max})$ or $\Delta_N(<z_{\max})$. As examples, we consider three representative CDM models (Table 1) whose fluctuation amplitude σ_8 is normalized so as to reproduce the abundances of clusters of galaxies (e.g., Kitayama & Suto 1997; Kitayama, Sasaki & Suto 1997). The results are displayed on a series of figures. Figure 1 shows how α is related to the comoving smoothing

length R in these models. Figure 2 displays $\Delta_N(R;z)$ as a function of z, and finally Figure 3 plots $\Delta_N(R;z)$ against R.

The general appearance of the figures suggests that the light-cone effect is a fairly robust feature, although its details depend on the model. In all cases SCDM appears to give the strongest effect, while for OCDM and LCDM it is slightly less pronounced. Nevertheless the difference is fairly small and qualitatively all models behave similarly. Note also, that the magnitude of the correction depends on the order N, and, in accord with intuition, it is monotonically increasing for higher order. As expected, the light-cone effect becomes larger as $z_{\rm max}$ increases, which can be seen in Figure 2. Although the correction is relatively small for shallow surveys with $z \lesssim 0.2$ samples, $\Delta_N(R;<z_{\rm max})$ becomes $\gtrsim 10\%$ in nonlinear scales ($R \sim 1h^{-1}$Mpc). In SCDM, for instance, $\Delta_N(R;<z_{\rm max})$ exceeds unity for $N \geq 6$ for the entire dynamic range plotted. Furthermore Figure 3 indicates that even if the hierarchical ansatz is correct, i.e., $S_N(R;z)$ is independent of R, the light-cone effect should generate apparent scale-dependence, since the correction behaves differently at different scales at a given redshift.

The future SDSS will be able to measure the moments of the galaxy density field with unprecedented accuracy. Unless unforeseen systematics exists, it will determine them with less than a few percent error for $N \leq 3$ and 10% for $N = 4$ between 1 and $50h^{-1}$Mpc (see Colombi, Szapudi, & Szalay 1997 for details). According to Figures 2 and 3, the light cone effect will be much smaller than these errors, or at most of the same order, depending on the scales and models. The correction could be potentially non-negligible only at the smallest scales. A similar conclusion can probably be drawn about the 2dF survey. On the other hands, for future deep surveys, which should aim at smaller scales especially if carried out by the Next-Generation Space Telescope, our calculations will be of utmost importance. According to Figure 3 the required correction can range from up to unity for S_3 through factors of few for S_6 to factors of hundred for S_{10}.

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Table 1: CDM model parameters

Model	Ω_0	λ_0	h	σ_8
SCDM	1.0	0.0	0.5	0.6
OCDM	0.45	0.0	0.7	0.8
LCDM	0.3	0.7	0.7	1.0

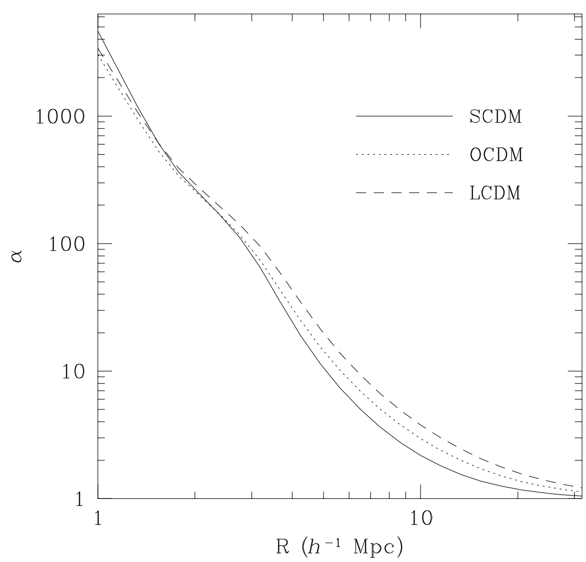

Fig. 1.— $\alpha(R)$ is plotted against $\log_{10} R(1h^{-1}\mathrm{Mpc})$ for SCDM (solid line), OCDM (thin dotted line), and LCDM (thick dotted line) summarized in Table 1.

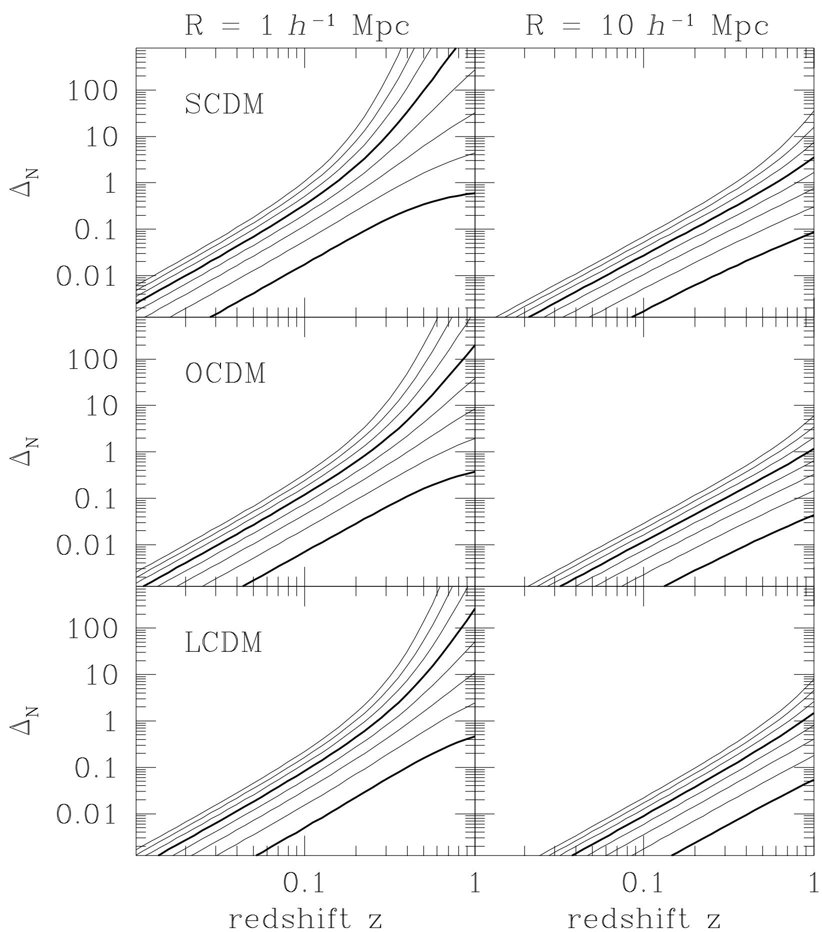

Fig. 2.— $\log_{10}\Delta_N(R;z)$ are shown as functions of $\log_{10} z$ at $R = 1h^{-1}$Mpc (*left panels*) and $10h^{-1}$Mpc (*right panels*); SCDM (*top panels*), OCDM (*middle panels*), and LCDM (*bottom panels*). The family of curves display different orders from $N = 3 \ldots N = 10$ monotonically upward; for $N = 3$, and $N = 7$ is plotted with thick lines for orientation.

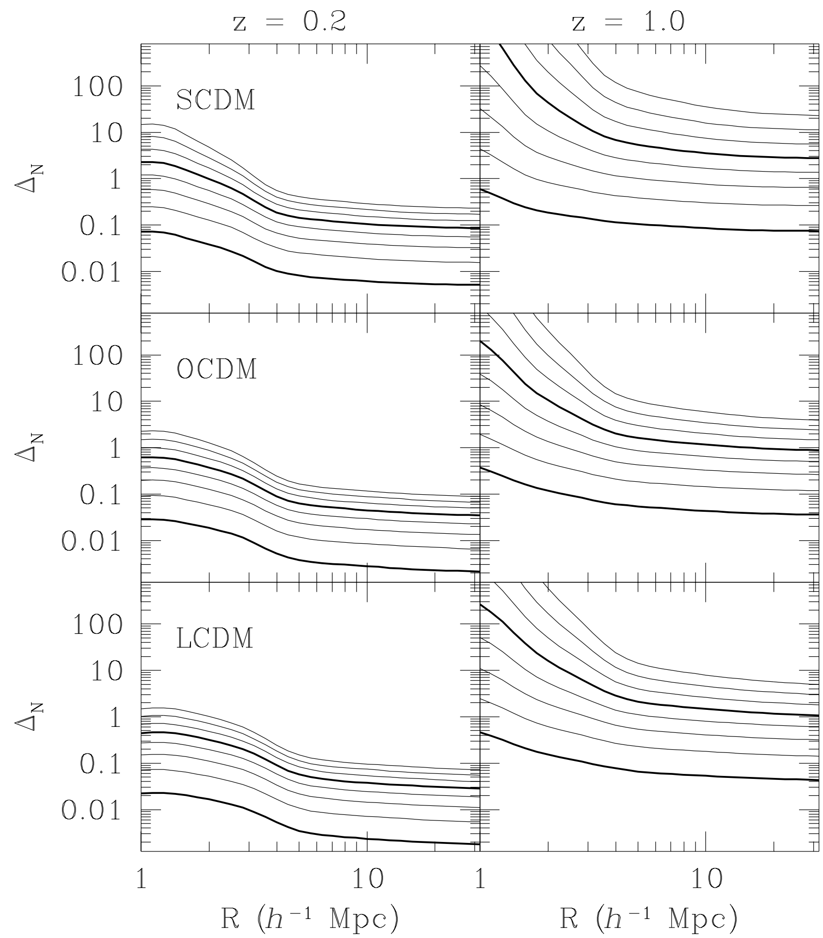

Fig. 3.— $\log_{10}\Delta_N(R;z)$ are displayed as functions of $\log_{10}R$ at $z = 0.2$ (*left panels*) and 1.0 (*right panels*); SCDM (*top panels*), OCDM (*middle panels*), and LCDM (*bottom panels*). The family of curves is the same as for Fig.2.